DSP

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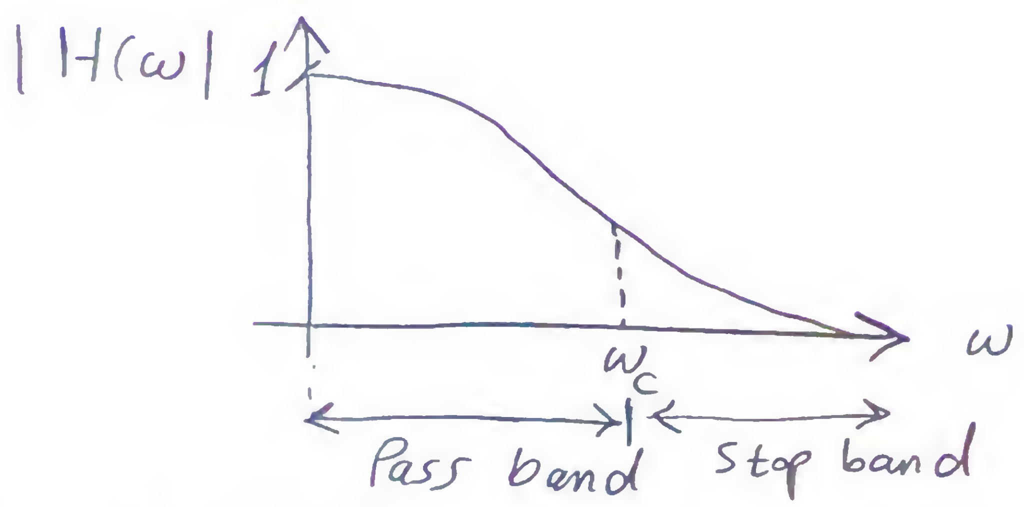
Analog Filter design :-

Digital Filters are discrete time systems that make operations related to Freq. such as:-

- Low pass
- high pass
- Band pass
- Band stop

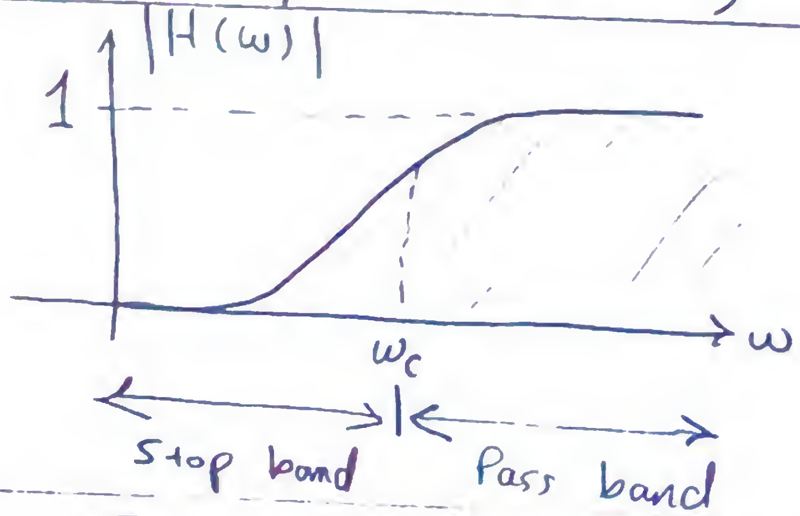
* types of Filters

① Low pass filter (LPF)

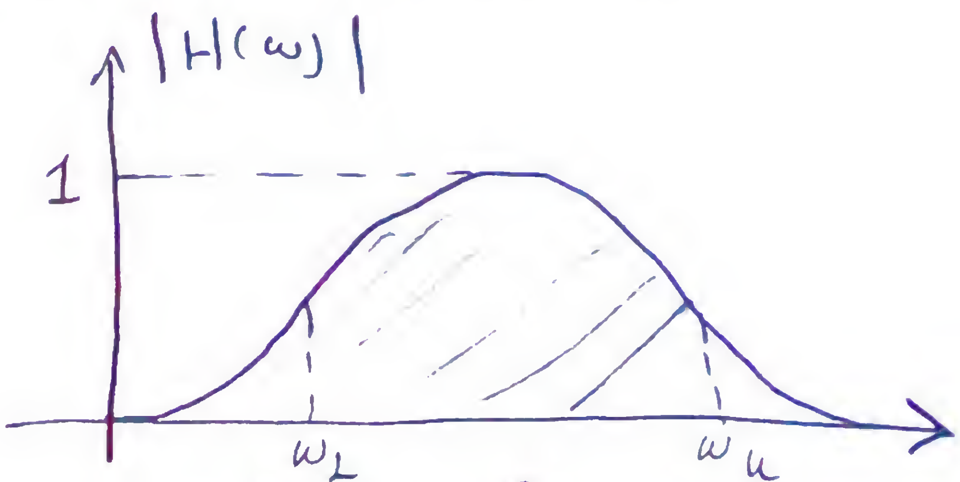


w_c → cut off freq (rad/sec)

② High Pass filter (HPF)



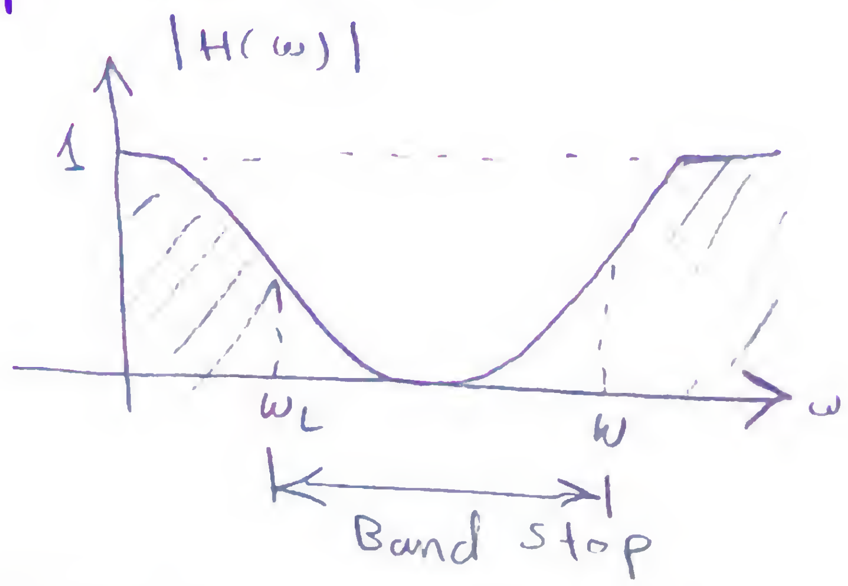
③ Band Pass Filter (BPF)



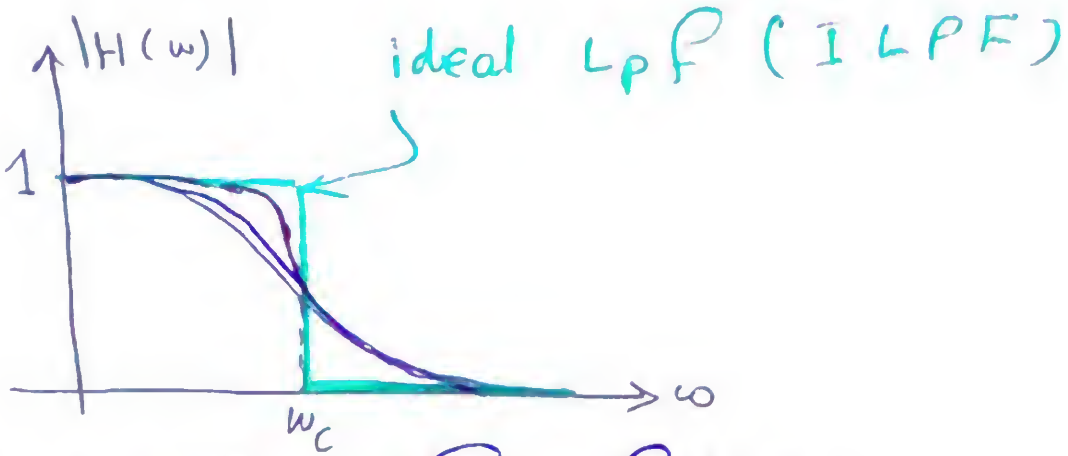
w_L → Lower cut off frequency.

w_u → upper cut off frequency.

[4] Band Stop filter (BSF)

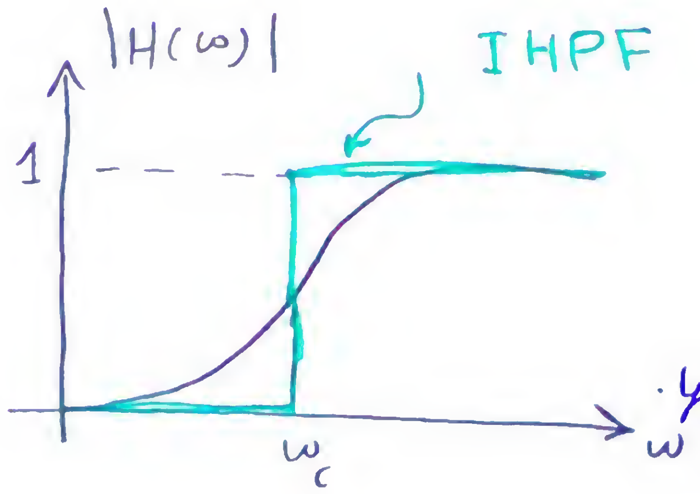


[1] LPF



$H(\omega) \Rightarrow$ T. F. for filter

[2] HPF



نعمل تقريب لل ideal filters

وأعمل بناء عليه Digital filter

Approximation methods for Analog filter design :-

1. Butter worth filter ✓✓
2. Elliptic filter
3. Bessel filter
4. chebyshev filter

The specs. required for design:-

① cut off freq. (w_c, w_L, w_u)

② order of the filter

③ type of filter → LPF, HPF, BPF, BSF

[1] Butter worth filter Design:-

The design is to find T.F. for the required filter, then we can implement it (H/w or S/w)

* The Design is made for LPF, then we can convert it to the required type

* Design for LPF (using Butter worth method):-

- The filter T.F. :-

$$H(s) = \frac{w_c^N}{(S-P_0)(S-P_1)\cdots(S-P_{N-1})}$$

$N \longrightarrow$ the order of the filter

$w_c \longrightarrow$ cut off frequency (rad/sec)

$$P_k = w_c \, e^{j(N+2K+1)\pi/2N}$$

$k = 0, 1, \cdots N-1$

Ex1-

$w_c = 0.725$ rad/sec , $N = 2$

$$H(s) = \frac{(0.725)^2}{(s-P_0)(s-P_1)}$$

$$P_0 = (0.725)\, e^{j(2+0+1)\frac{\pi}{4}} ; \ 135^\circ$$

$$= 0.725 \left[\cos(135) + j \sin(135)\right]$$

$K=0 \rightarrow P_0 = -0.513 + j\,0.513$

$K=1 \rightarrow P_1 = (0.725)\, e^{j(2+2+1)\frac{\pi}{4}}$ 225°

$P_1 = -0.513 - j\,0.513$

$$H(s) = \frac{(0.725)^2}{(\underbrace{s+0.513}_{x} - j\underbrace{0.513}_{y})(\underbrace{s+0.513}_{x} + j\underbrace{0.513}_{y})}$$

$$H(s) = \frac{(0.725)^2}{(s+0.513)^2 + (0.513)^2}$$

$$\boxed{H(s) = \frac{0.526}{s^2 + 1.026 + 0.526}}$$

Normalized LPF = NLPF

A NLPF is an LPF with $w_c = 1$ rad/sec

$N = 1 \Rightarrow H(s) = \dfrac{1}{(s - P_0)}$

$P_0 = e^{j(1+0+1)\pi/2} = e^{j\pi}$

$= \underbrace{\cos(180)}_{-1} + j\,\underbrace{\sin(180)}_{0} = -1$

$P_K = w_c\, e^{j(N+2K+1)\pi/2N}$

$w_c = 1$

$P_K = e^{j(N+2K+1)\pi/2N}$

$$H(s)\Big|_{NLPF} = \frac{1}{s+1}$$

$N = 2$

$$\Rightarrow H(s)\Big|_{NLPF} = \frac{1}{(s-P_0)(s-P_1)}$$

$$P_0 = e^{j(2\cdot 0+1)\pi/4} \leftarrow 135^\circ$$

$$= \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$P_1 = e^{j(2\cdot 2+1)\pi/4} \angle 225^\circ$$

$$= \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$H(s)\Big|_{NLPF} = \frac{1}{\left(s+\frac{-1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)\left(s+\frac{-1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{1}{\left(s+\frac{1}{\sqrt{2}}\right)^2+\frac{1}{2}}$$

$$= \frac{1}{s^2+\frac{2}{\sqrt{2}}s+\frac{1}{2}+\frac{1}{2}}$$

$$= \frac{1}{s^2+\sqrt{2}s+1}$$

$\therefore$ NLPF (Butterworth)

$N=1 \longrightarrow H(s) = \frac{1}{s+1}$

$N=2 \longrightarrow H(s) = \frac{1}{s^2+\sqrt{2}s+1}$

$N=3 \longrightarrow H(s) = \frac{1}{(s+1)(s^2+s+1)}$

Convert from NLPF to any other type :-

NLPF

① LPF (with $\omega_c \neq 1$)

$$s \to \frac{s}{\omega_c}$$

$$H(s)\Big|_{LPF} = H(s)\Big|_{\substack{NLPF \\ s \to \frac{s}{\omega_c}}}$$

② HPF (with $\omega_c \neq 1$)

$$s \to \frac{\omega_c}{s}$$

$$H(s)\Big|_{HPF} = H(s)\Big|_{\substack{NLPF \\ s \to \frac{\omega_c}{s}}}$$

③ BPF

$$s \to \frac{s^2 + \omega_u \omega_L}{s(\omega_u - \omega_L)}$$

④ BSF

$$s \to \frac{s(\omega_u - \omega_L)}{s^2 + \omega_u \omega_L}$$

Ex. Design HPF with $\omega_c = 22$ rad/sec & $N = 3$

$$H(s)\Big|_{\substack{NLPF \\ N=3}} = \frac{1}{(s+1)(s^2+s+1)}$$

$$H(s)\Big|_{\substack{HPF \\ \omega_c = 22 \text{ rad/sec} \\ s \to \frac{22}{s}}} = \frac{1}{\left(\frac{22}{s}+1\right)\left(\left(\frac{22}{s}\right)^2 + \frac{22}{s} + 1\right)}$$

$$= \frac{s^3}{(22+s)\left((22)^2 + 22s + s^2\right)}$$

Design Digital Filters :-

* The design is made in analog domain and then use bilinear transformation to convert the design into Digital domain.

$$\boxed{NLPF} \longrightarrow \boxed{AF} \xrightarrow[\text{Bilinear Transformation}]{} \boxed{\text{digital filter}}$$

$$s \longrightarrow z$$

Bilinear Transformation is a method used to approximate the mapping from s-domain to z-domain

$$S = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$z = e^{Ts} = e^{T/2 \times s} \cdot e^{T/2 \times s}$$

الإثبات غير مطلوب

$$= \frac{e^{\frac{T}{2}s}}{e^{-T/2\, s}} = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$$

* From taylor:

$$e^{x} = 1 + x + \underbrace{\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots}_{\to 0}$$

assume $x \leq 1 \Rightarrow e^{x} = 1 + x$

$$z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$$

$$z - \frac{T}{2}sz = 1 + \frac{T}{2}s$$

$$z - 1 = \frac{T}{2}sz + \frac{T}{2}s$$

$$z - 1 = (z+1)\frac{T}{2}s \Rightarrow s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

* design specs are given in digital domain

ω_{c_D} , ω_{u_D} , ω_{L_D} , N, Type, T

D → digital domain

هحتاج أرجع لل Analog عشان أعمل Mapping لل Digital Domain

ω_c , ω_u , ω_L Analog domain

$$\omega'_A = \frac{2}{T} \tan\left(\frac{\omega_D T}{2}\right)$$

Analog

Digital

Ex: Design Band pass filter with N=1 with the following specs:

* $f_{L_D} = 2.4$ KHZ (Lower cutoff freq = ✓)
* $f_{u_D} = 2.6$ KHz (upper cutoff freq = ✓)
* $f_s = 8$ KHz (Sampling freq. = ✓)

$T = \frac{1}{f_s}$

الحسابات بال radian

1. Convert ω_{L_D}, ω_{u_D} ⟹ ω_L, ω_u

Digital → Analog

$$\omega_L = \frac{2}{\frac{1}{8\times10^3}} \tan\left(\frac{2.4\times10^3\times2\pi}{2\times8\times10^3}\right)$$

rad

$T = \frac{1}{8\times10^3}$ sec

$= 22022.1107$

$\omega_{L_D} = 2\pi f_D$

$= 2\pi\times2.4$

$$\omega_u = \frac{2}{1/8\times10^3} \tan\left(\frac{2\pi\times2.6\times10^3}{2\times8\times10^3}\right)$$

$= 26109.63$ rad/sec

$\boxed{2}$ NLPF with $N=1$

$$H(s)\Big|_{\substack{NLPF \\ N=1}} = \frac{1}{S+1}$$

$$H(s)\Big|_{\substack{BPF \\ \text{with } \omega_L, \omega_u}} = H(s)\Big|_{\substack{NLPF \\ S \longrightarrow \frac{S^2+\omega_u\omega_L}{S(\omega_u-\omega_L)}}}$$

$$H(s) = \frac{(\omega_u-\omega_L)\,s}{s^2+(\omega_u-\omega_L)\,s+\omega_u\omega_L}$$

$$H(s)\Big|_{ABPF} = \frac{4087.516\ S}{S^2+4087.51\,S+574989096.1}$$

$\boxed{3}$ using Bilinear Transformation (B.T)

$$S \longrightarrow \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$\Rightarrow H(z)\Big|_{DBPF} = H(s)\Big|_{\substack{ABPF \\ S \longrightarrow 2f_s\left(\frac{z-1}{z+1}\right)}}$$

$$H(z) = \frac{0.0728z^2-0.0728}{z^2+0.7118z+0.854}$$

Ex: design a first order Digital LPF with $\omega_c = 30\pi$ rad/sec and Sampling time $T = 1/90$ sec

given $\left.\begin{array}{l} \omega_{c_0} = 30\pi \text{ rad/sec} \\ T = \frac{1}{90} \text{ sec} \end{array}\right\}$ Digital domain

① obtain $w_{c_{(A)}}$ Analog

$$w_c = \frac{2}{T} \tan\left(\frac{w_{c_D} T}{2}\right)$$

$$= 180 \tan\left(\frac{30\pi}{90 \times 2} \times \frac{180}{\pi}\right) \simeq 103.923 \text{ rad/sec}$$

$\boxed{2}$ $N = 1$, NLPF

$$H(s)\Big|_{\substack{NLPF \\ N=1}} = \frac{1}{s+1}$$

$$H(s)\Big|_{\substack{LPF \\ N=1 \\ w_c = 103.923}} = H(s)\Big|_{\substack{NLPF \\ s \to \frac{s}{w_c}}} = \frac{1}{\frac{s}{w_c} + 1} = \frac{w_c}{s + w_c}$$

$$= \frac{103.923}{s + 103.923}$$

$\boxed{3}$ Using B.T.

$$H(z) = H(s)\Big|_{LPF} = \frac{103.923}{180\left(\frac{z-1}{z+1}\right) + 103.923}$$

$$s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$H(z)\Big|_{DLPF} = \frac{0.366(z+1)}{z - 0.2679}$$